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MATHEMATICAL MODELING OF THE VAPOUR PERMEABILITY OF TEXTILE MATERIALS

Daniela FARIMA[1], Ioan IACOB[2], Georgios PRINIOTAKIS[3]

[1,2] „"Gheorghe Asachi" Technical University, Bd.Mangeron, Iasi, Romania, d_farima@yahoo.com; iiacob@tex.tuiasi.ro

[3] University of West Attica, Thivon & P. Ralli Ave.GR-122 44, Egaleo, Greece, E-mail: gprin@puas.gr

Corresponding author: Daniela Farima, d_farima@yahoo.com

Abstract: *The current research is based on experimental values regarding the vapor permeability of knitted knitted fabrics made of different yarns. The objective of the research is to define the transfer of moisture by variables which establish the existence of a functional interdependence. For this purpose mathematical modeling of the respective transfer is required, finding the type of function that expressesthe best the correlation between the variables and the dynamic evolution of the vapor transfer as a whole. The paper aims to highlight the correlation between the structure parameters of the double layer knits and their vapor permeability. The mathematical modeling of vapor permeability through these knits consists of two stages: obtaining the mathematical model and testing it. The mathematical model used in the experimental data processing, allowed the quantitative expression of the thermophysiological comfort characteristic, depending on the structure parameters of the analyzed knits. Based on the mathematical model, graphical representations were obtained by linear and spatial interpolation, adapted to concrete situations. These allow information to be obtained regarding the modification of the vapor permeability capacity according to the structural parameters involved in its mathematical model. The mathematical model obtained may be the starting point for further research, in which to obtain knits with new features and destinations.*

Key words: *vapour permeability, knits, structure parameters, regression equation, correlation coefficients*

1.INTRODUCTION

The mathematical function can be linear or nonlinear and is defined by the regression analysis, which must necessarily be completed with the correlation analysis, that obtains the quantitative expression of the strict dependence of one variable on the other, dependence given by the value of the correlation coefficient [1]. The quality of the regression equations depends on the experimental data on the one hand, and on the other hand on the used method to obtain the coefficients of these equations.

One of the most representativ statistical mathematical models is the regression model, which is generally used in cases when the solution of the theoretical problem encounters difficulties because of the complexity the functional interdependencies in term of theoretical analyses or mathematical formulation.

This paper aims to highlight the correlation between the structure parameters of six variants of double layer knit V_1, V_2, V_3, V_4, V_5, V_6 (cotton threads on the back and polypropylene threads on the front) and their the vapour permeability C_{vap} [%].The mathematical modeling of vapour permeability consists two stages: obtaining of the mathematical model and testing it.

2.OBTAINING THE MATHEMATICAL MODEL

The initial data required to develop regression models for the research results are:
- independent variables denoted by X_i (structure parameters of layered knits: surface density on the back of the knits $X_1 = D_{ss}$ [N_{os} / rap], horizontal densities ratio $X_2 = D_{of} / D_{os}$, verticality densities ratio $X_3 = D_{vf} / D_{Vs}$; thickness $X_4 = g_t$ [mm], surface area mass $X_5 = M$ [g / m^2], number of patented loops $X_6 = N_{op}$ / rap, porosity $X_7 = P_z$ [%] %]) (table 2);
-dependent variable: vapour permeability capacity C_{vap}. [%] denoted by y (table 1).

Table 1:*The values of vapour permeability capacity C_{vap} [%]*

The thermo-physiological characteristic y	Knit variant V_i					
	V_1	V_2	V_3	V_4	V_5	V_6
$y=C_{vap\ Vi}$[%]	36,2	42,1	40.1	33,1	40,11	36,5

Table 2 :*The structures parameters values*

Structure parameters of knit variants xi	Knit variant V_i					
	V_1	V_2	V_3	V_4	V_5	V_6
$x_1=D_{ss\ Vi}$[Nos/rap]	5680	4880	5246	5796	5247	5325
$x_2=D_{of}/D_{os\ Vi}$	0,971	0,967	0,950	0,968	0,934	0,957
$x_3=D_{vf}/D_{vs\ Vi}$	1,025	1,037	1,023	1,021	1,026	1,034
$x_4=g_{t\ Vi}$[mm]	1,39	1,24	1,20	1,11	1,42	1,54
$x_5=M_{Vi}$[g/m^2]	245	242	235	236	253	277
$x_6=N_{op}/rap_{Vi}$	2	2	4	8	94	42
$x_7=P_{zVi}$ [%]	79,72	78,85	80,72	76,21	81,50	82,07
$x_8=P_{PP\ Vi}$[%]	53,14	57,97	55,79	52,46	51,76	50,37

In the correlation analysis we determined the simple correlation coefficients noted with ryxi. Their values are shown in table 3, which also that in the mathematical models were included only the structure parameters with the simple correlation coeffcient *ryxi* > 0.5 un significant at Student test for the other cases [2].

Table 3: *The values of the simple correlation coefficients*

The type of correlation yi=f(xi)	The value of the simple correlation coefficient ryxi	The significance of the correlation
$C_{vap\ Vi}=f(D_{ss\ Vi})$	-0,92705270472449	Significant
$C_{vap\ Vi}=f(D_{of}/D_{os\ Vi})$	-0,43378908369164	Insignificant
$C_{vap\ Vi}=f(D_{vf}/D_{vs\ Vi})$	0,49961603576932	Insignificant
$C_{vap\ Vi}=f(g_{t\ Vi})$	0,07686474297386	Insignificant
$C_{vap\ Vi}=f(M_{Vi})$	-0,07946529996672	Insignificant
$C_{vap\ Vi}=f(N_{op}/rap_{Vi})$	0,18041475920435	Insignificant
$C_{vap\ Vi}=f(P_{zVi})$	0,45093868277429	Insignificant
$C_{vap\ Vi}=f(P_{PP\ Vi})$	0,64266343492296	Significant

It is assumed that if $t_c > t_T$ ($t_T = t_{0,05}$; n = n-2) there is a link between the two variables; Otherwise, the correlation link is missing. The multiple regression equation was established by testing the signifiance of the simple correlation coefficient only for yhose structure parameters proved to be signifiant (table 4).

The multiple correlation coefficient R, of which value expresses the total correlation between the independent variables and the dependent variable, used the values of the simple correlation coefficients. In case of perfect adjustment the value of the multiple correlation coefficient R=1 (table 4).

Multiple determination coefficient R^2 (table 4) expresses that part of the variation of the moisture permeability characteristic, which can be attributed to the group of knot structure parameters that enter the mathematical model. The value $(1-R^2)$ indicates the part that can be attributed to the experimental error, the deviation from the assumed linearity and the dispersion of the experimental data.

One of the most difficult problems to be solved by obtaining those structural parameters that significantly influence the vapor permeability characteristic C_{vap} [%] is their interpretation. The most commonly used method is multiple regression analysis, which aims to establish a regression equation that exists between the comfort feature and multiple structural parameters.

In order to check the significance of the multiple correlation coefficient, the Fischer criterion was used; if $F_c > F_T$ ($F_T = F_{0,05}$; $\nu_1 = k$; $\nu_2 = n_{k-1}$) is assumed the existence of a correlation between the dependent variable Y (vapour permeability) and the independent variables X_1, X_2,......X_k (the structure parameters, k = 8) with a 95% probability. The concrete form of the multiple regression equation found for vapor permeability capacity y = C_{vap} [%] is $y = C_{vap} = b_0 + b_1 D_{ss} + b_2 P_{PP} + b_3 P^2_{PP}$ and is presented in fig.1.

In table 4 are shown the limits of the confidence intervals for the real values of the of the regression equation coefficients by using the Student distribution for a 95% statistical certainty.

Table 4: *The limits limitele intervalelor of the confidence intervals R, R^2*

The concrete form of the regression equation	The value of the equation coefficients of regression	The interval limits of confidence for regression coefficients	R	R^2
y=Cvap.=b_0+b_1Dss+b2PPP+b3P²PP	b_0= - 711.3549 b_1= 30.4155 b_3= - 0.2814	-1674.32÷251.61 -0.0208÷0.05495 -6.671÷67.750 -0.626÷0.0638	0.991	0.982

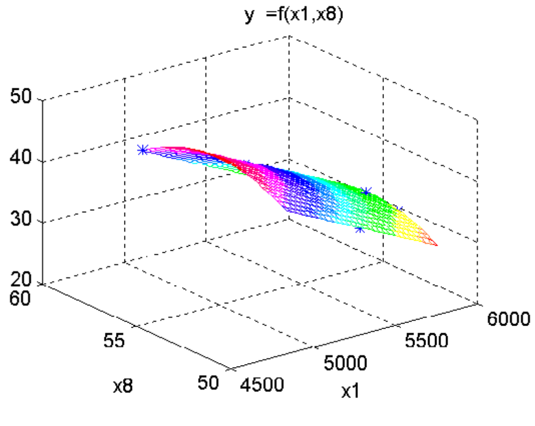

Fig.1: *$y = C_{vap.} = b_0 + b_1 Dss + b_2 P_{PP} + b_3 P^2_{PP}$*

3.THE STATISTICAL TESTING OF THE MATHEMATICAL MODEL

The extent of which the mathematical model reflects the actual situation from which it started can be verified by calculating the residues attached to the regression, residues calculated as deviations from the values observed for y_{oi} and the estimated y_{ei} of the vapor permeability capacity C_{vap} [%] (equation 1) [3]:

$$r_i = y_{0i} - y_{ei} = (y_{0i} - \bar{y}) - (y_{ei} - \bar{y}) \quad (1)$$

The residual values obtained, their maximum values for the vapour permeability capacity C_{vap}, as well as the confidence intervals for the residual values are presented in table 5 and fig. 2. The significance threshold considered is $\alpha = 0{,}05$.

Table 5: *Tthe confidence intervals for residues and the maximum residue value over the observed value $C_{vap.Vi}$*

Knit variant	The value of the residue $r_{Cvap.Vi}$	The maximum residue value compared to the observed value [%]	The limits of confidence intervals for residues
V_1	-0,4077	1,946	-2,2820÷1,4664
V_2	0,6910		-0,56581÷9480
V_3	-0,1173		-1,8634÷1,6288
V_4	-0,4863		-2,4552÷1,4824
V_5	0,1386		-2,0042÷2,2814
V_6	0,1817		-0,7731÷1,1366

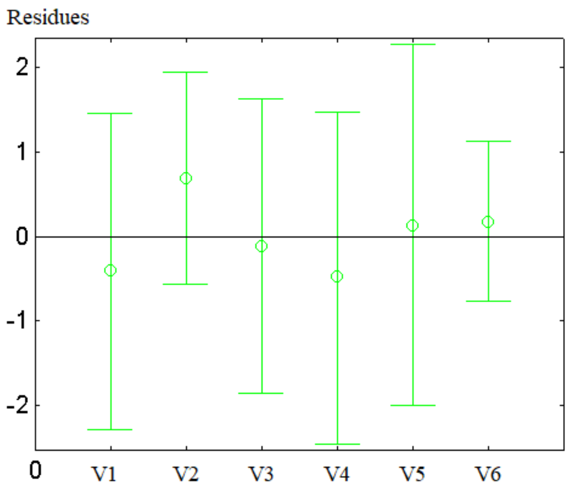

Fig. 2: *Graphic representation of the C_{vap} on knitting variants*

Small residues reflect a better adjustment of experimental data, but setting a criterion indicating how small the residue should be for regression, in order to be accepted, is a difficult issue. The acceptance of a regression cannot be admetted by using only the residue's size. For this purpose, it was calculated the multiplication correlation coefficient R whose value expresses the total correlation between the group of independent variables and the dependent variable and the value of the coefficient of multiple determination R2 (table 4).

On the based of the residue calculation equation [5], it was possible to express the dispersion analysis presented in table 6.

A first clue on the mathematical model adequacy is obtained by analyzing the squares sums for the regression S_{Preg} and the residues S_{Prez} [3] (equation 2,3).

$$S_{Preg} = \sum_{i=1}^{n} (y_{ei} - y_{mediu})^2 \qquad (2)$$

$$S_{Prez} = \sum_{i=1}^{n} (y_{oi} - y_{ei})^2 \qquad (3)$$

The smaller the S_{Prez}. comparing to S_{Preg} is, the better the mathematical model adjusting.In this case it can be stated that the variation in the survey data is in the range of the estimated data, so the estimated data "represents" the data determined experimentally.The data in table 6 of the dispersion analysis indicates that the mathematical model adjustment is acceptable. A matter of great practical importance is the prediction of the estimated value y_0 for an x_0 that was not taken into account when adjusting the mathematical model. This situation occurs when the mathematical model is used to study the real phenomenon outside the values considered in modeling. As an indicator of the quality of regression, the most common test was the Fisher F.

Table 6*: Dispersion analysis of multiple regression equations*

Source of variation	Degrees of freedom	Squares sum	Average squares	F	F_T
Cvap.=b0+b1Dss+b2PPP+b3P¸PP	3	54,2236	18,074		
rmax.=1,946[%]	2	0,9464	0,4732	95	19,2
Total	5	55,1700	-		

In the literature [4] there it is a recommended to use a regression equation for prediction purposes if the F statistic is more than 4 times higher than the table value „F_T". The probability of invalidation of the regression capacity of the permeability capacity at vapor C_{vap} [%] is 0.02562.

4. INTERPRETATION OF MATHEMATICAL MODELS

In the mathematical model describing the vaporization capacity of water C_{vap} ($y = C_{vap} = b_0 + b_1D_{ss} + b_2P_{PP} + b_3P_{2PP}$ or $y = C_{vap} = b_0 + b_1x_1 + b_2x_8 + b_3x^2_8$ or $y = C_{vap} = -711.35, 49 - 0.01315x_1 + 30.4155x8 - 0.2814x^2_8$), the structure parameters have the following experimentally determined values: x1: [4880; 5246; 5247; 5325; 5680; 5796]; x_8: [50.37; 51. 76; 52.46, 53.14; 55.79; 57.97] [5].

If the percentage of polypropylene x_8 = 51.5% is constant, y = f (x_1) (fig.3), can be calculated.

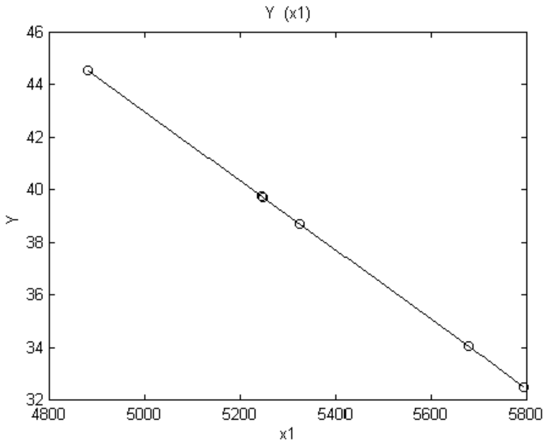

Fig. 3: *$y=f(x_1)$; x_8- constant*

When we want to obtain a cotton and polypropylene layered knit fabric with a surface area on the back Dss = x1 = 5059 [Nos / rap] we can determine the vaporization capacity C_{vap} . = y = f (x_8) by using the linear interpolation [6].

5.CONCLUSIONS

The use of mathematical model in the processing of the experimantal data for the vapour permeability capacity cvap allowed to express the quantity of the thermo-physiological comfort characterisic by using the structure parameters of the analyzed knits.

The obtained mathematical model can be a starting point for further research in which:

- other types of raw materials or the same as in the present paper, that with different density lenth can be used in order to obtain the knitted fabrics;
- it can modify the structure parameters by modifyng the technolocal parameters;
- different finishing treatments can be applied to knitted fabrics, etc.

Based on the mathematical model of graphical representation obtained by linear and spatial interpolation, adapted to concrete situations, it is possible to obtain information regarding the modification of the comfort characteristic C_{vap} [%], depending on the structural parameters involved in its mathematical model.

By expanding the research, it is possible to use the experimental programming for predicting the studied phenomena.

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